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INVESTIGATION OF SYSTEMS WEAKENED BY KINKED CRACKS

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Abstract: structural strength of aircraft is a key aspect of flight safety. Hidden defects in the material significantly affect its strength under various loads. The crack growth rate and direction, and the crack growth threshold load (stress intensity factor) affect the strength of the damaged material. This study investigates a 3D elastic structure weakened by a system of flat cracks and a kinked crack. The numerical method used was the boundary element method, specifically, the displacement discontinuity method. The code was developed with C++. The results were compared against the available analytic results. The behavior of cracks under bending and a range of loading conditions was studied.

Keywords: 3D space, elastic medium, crack, stress intensity factor, boundary element method, displacement discontinuity method.

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ИЗУЧЕНИЕ СИСТЕМ, ОСЛАБЛЕННЫХ ТРЕЩИНАМИ С ИЗЛОМОМ

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Аннотация: прочность летательных аппаратов любых типов — важнейший вопрос безопасности полетов. Наличие скрытых дефектов в материале существенно влияет на прочность при различных нагрузках. Важными характеристиками прочности материалов с дефектами являются скорость и направление роста трещины, а также величина критической нагрузки (коэффициента интенсивности напряжений), при которой начинается рост трещины. В данной работе исследуется трехмерная упругая среда, ослабленная системой плоских трещин и одной трещиной с изгибом. В качестве численного метода был выбран метод граничных элементов, а именно метод разрывных перемещений. Код реализован на C++. Было проведено сравнение с известными аналитическими результатами. Изучено поведение трещин при изгибе при различных нагрузках.

Ключевые слова: трёхмерное пространство, упругая среда, трещина, коэффициент интенсивности напряжений, метод граничных элементов, метод разрывных смещений.

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Introduction

Structures can experience planned and random external loads that affect their safety [1]. The study of the strength of aircraft structures is a complex process, which can be considered from different perspectives. The study of strength under different types of loads is presented in [2, 5], and the strength under high-temperature loads is discussed in [3]. There have also been studies in which the spatial failure of the membrane has been investigated [6]. The problem of asteroid destruction during their interaction with impact or explosive missiles is also relevant [4]. Data on the presence of micro defects inside asteroids help us to understand where the heterogeneities are located. This allows us to predict where the impactor should be directed to blow up the asteroid, getting the maximum effect at minimum cost. The development of such missions requires predictive numerical modeling, which relies heavily on the science of strength of materials and structures, particularly linear fracture mechanics. The foundations of this theory were developed in [7, 8]. The primary cause of fracture is the presence of defects in the material in the form of so-called cracks, which are simulated by the displacement field jump on a certain part of the surface. For an elastic medium, this leads to the appearance of features at the crack boundary. When approaching the crack boundary, the stresses tend to infinity, i.e. the concentration of stresses occurs in a sufficiently small vicinity of the boundary. Since the existence of infinite stresses in real materials is impossible, a region of irreversible plastic deformations occurs near the crack edges. Nevertheless, in cases where the size of this field is small compared to the size of the crack itself, the applicability of the crack growth criteria is based on the analysis of the elastic solution obtained [9–14]. Linear fracture mechanics has developed rapidly and is currently one of the main tools for assessing the strength of materials with defects. A sufficiently complete picture of the results obtained in this field is given by the reviews [14–18].

We use the three fundamental solutions of the theory of elasticity about the discontinuity of the three components of the displacement vector on the surface of the boundary element as the basic functions of the solution decomposition. The general solution of the entire problem is represented as a sum with indefinite coefficients, i.e., as a finite series of analytically defined basis functions. The coefficients of the series are determined by the collocation method on the boundary (the boundary conditions are fulfilled only in the centers of boundary elements). This approach makes it possible to avoid calculating singular integrals that arise when using direct methods of boundary integral equations.

The advantage of the boundary element method is that only the fracture surface, modeling the fracture of the elastic medium, is broken into finite elements. This reduces the scale of the problem at the stage of its solution. Three independent analytical solutions are used for each element, in each of which one of the three displacement vector components suffers a discontinuity in the element. The solution to a particular boundary value problem is sought as a series with uncertain coefficients over the entire set of elements. Each element solution contributes to the displacement field and the stress field with a weight, which is the corresponding uncertain coefficient of the series. Fulfilling the specific boundary conditions leads to a system of linear equations after numerically determining the expansion coefficients. We have an analytical representation of the solution as a finite series within the domain. In terms of memory, we only need to save the found expansion coefficients, which will then allow us to find any desired characteristics at any point in the solution domain. This is important in terms of the ease of practical use of the resulting solution. Another important advantage of the proposed method is the possibility to solve any boundary value problem (stress problem, displacement problem, any mixed problem).

The disadvantage of the method is its weak mathematical reliability; therefore, a large amount of work is required to verify the reliability of the results. For this purpose, a comparison was made with the available analytical solutions of spatial problems, as well as with the known results of numerical solutions of fracture mechanics problems obtained by other numerical methods. The program codes have been implemented by the authors in C++. The main characteristic of linear fracture mechanics is the stress intensity coefficient at the crack edge (Fig. 1), which in the case of tensile strain in the direction normal to the crack plane is defined as

$$K_I = \lim_{s\to 0}\sqrt{2\pi s}\ \cdot \sigma_{zz}(s),$$

in the case of shear deformation in the crack plane along the normal to its edge is defined

$$K_{II} = \lim_{s\to 0}\sqrt{2\pi s}\ \cdot \sigma_{nz}(s),$$

and in the case of antiplane deformation (shear in the crack plane tangential to the edge) is determined.

$$K_{III} = \lim_{s\to 0}\sqrt{2\pi s}\ \cdot \sigma_{\tau z}(s).$$

This problem was solved numerically in the three-dimensional formulation for different radii. The comparison results allow us to speak about sufficient efficiency and satisfactory accuracy of the proposed method [22].

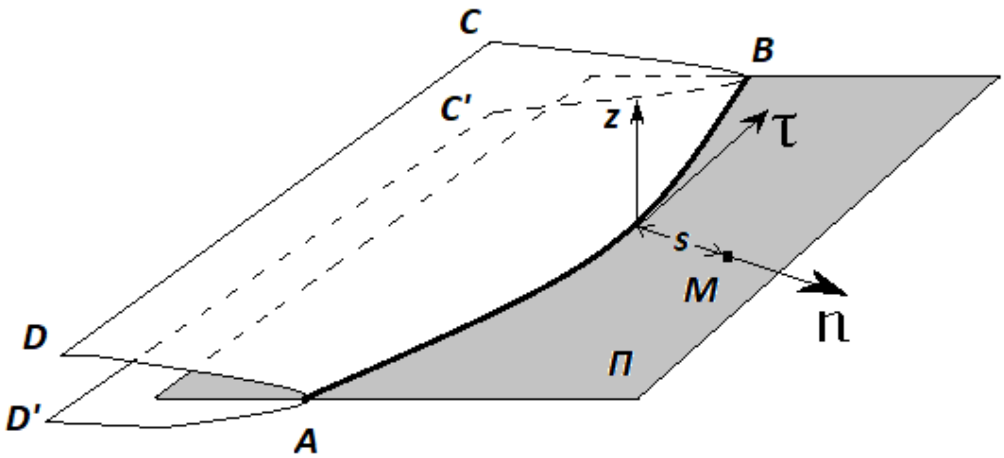


Figure 1. *Determination of the stress intensity coefficients for the cases of different types of deformations*

The written program was tested by comparing it with known analytical solutions [15–16], [19–21]. The comparison showed good qualitative and quantitative agreement with the available results of other authors. For example, a comparison was made with the solution for an axisymmetric crack in the form of a disk, which is under the action of internal pressure. In the cylindrical coordinate system r,φ,z (the crack corresponds to the disk $z= 0, 0 \le r \le R$) and in the given problem we have boundary conditions: $z= 0, 0 \le r \le R, \sigma_{zz}= -p, \sigma_{rz}= 0$. In the analytical solution for a circular crack the value K_I is equal to:

$$K_I = \frac{2}{\sqrt{\pi R}} \int_0^R \frac{r\sigma_{zz}(r)dr}{\sqrt{R^2 - r^2}}.$$

Circular fracture with fracture (fracture angle $30°$) is under internal pressure $p = 0.1$

Of some practical interest are cracks, the surface of which has a fracture (the angle between the planes of individual segments of the crack is different from zero). Such a geometric configuration can arise at the merger of two separate plane cracks. In linear elasticity theory, the presence of any jumps in the boundary conditions leads to peculiarities in the solution. Since the crack model itself (the discontinuity surface of the displacement field) carries a peculiarity of the solution, it is interesting to investigate the effect of a possible kink in its surface on the change in the stress distribution in the vicinity of the crack edge.

We consider an elastic medium weakened by a fracture crack. The crack is active, that is, it is loaded with internal pressure P, as shown in Fig. 2. Geometrically, the surface of the crack is two semicircles of the same radius, the planes of which are located at a given angle to each other. It is necessary to investigate in which direction the crack is most likely to grow.

The value of the J-integral of Cherepanov-Rice is taken as the main characteristic responsible for the possible crack growth. This integral is a combination of the squares of the stress intensity coefficients (Cherepanov and Rice) [11–13]:

$$J = \frac{1-\nu}{2\mu}\left(K_I^2 + K_{II}^2\right) + \frac{1}{2\mu}K_{III}^2.$$

Since in the geometric configuration under consideration the two halves of the crack are equal, the edge of that half of the crack, which is located in the plane xy(Fig. 2), has been studied. In the calculations, dimensionless quantities have been used: the length unit is the radius of half-circle of each half of the crack; stresses and pressure have been referred to the value 2μ, Poisson's ratio has been chosen to equal $\nu = 0.25$. The internal pressure is assumed to be $p = P/2\mu = 0.1$. Such an unrealistically large pressure has been chosen to make it possible to visualize the crack opening. The angle between the planes of two

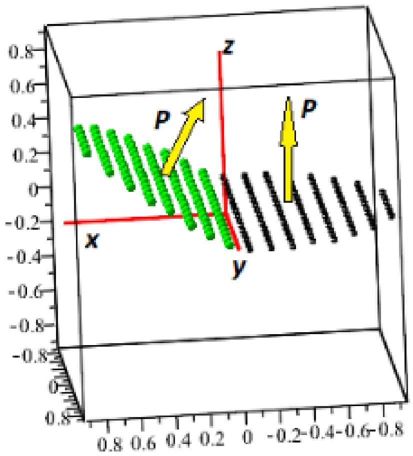
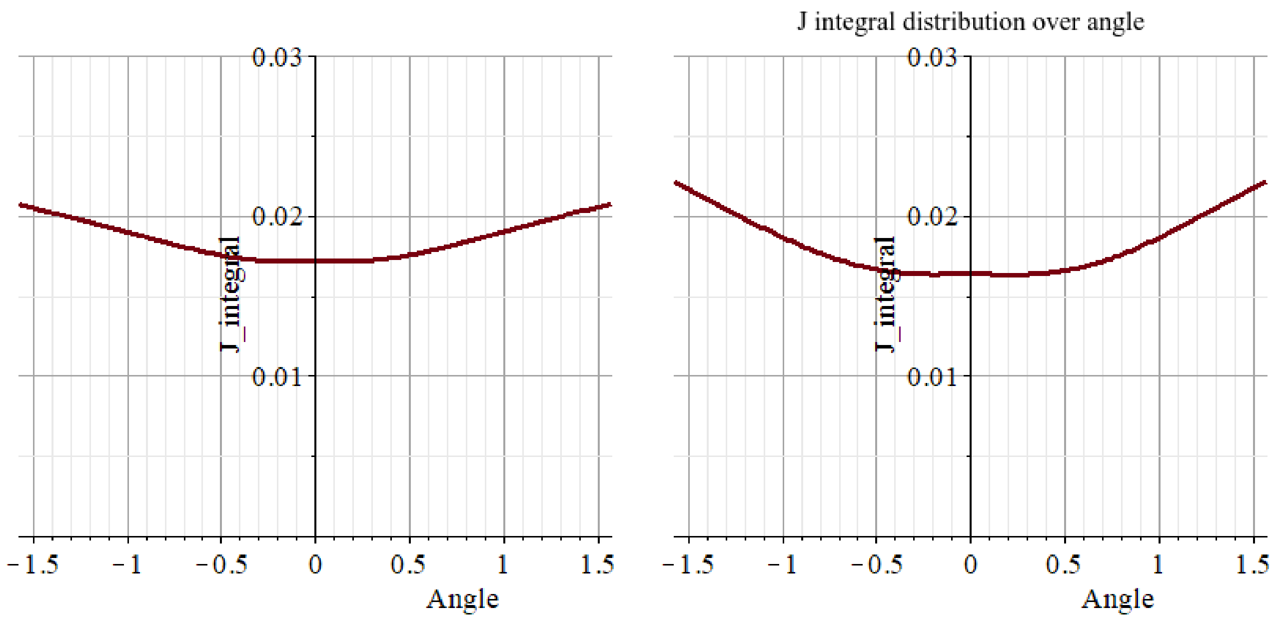


Figure 2. *Crack with fracture under internal pressure, directed along the normal to the banks of the crack*

planar semicircular segments of the crack is called a fracture angle. For comparison, the calculations for two fracture angles with values 30° and 60°.

Fig. 3 shows graphs of the value J vs. the angle. The angle is counted along the arc of the circular edge of the crack and varies within $[-\pi/2,\pi/2]$. The zero value of the angle corresponds to the point $x = -1, y = 0, z = 0$ (Fig. 2), the value of the angle $\pi/2-$ corresponds to a point with coordinates $x = 0, y = 1, z = 0$, the angle $-\pi/2$ corresponds to a point with coordinates $x = 0, y = -1, z = 0$. The last two points are on the fracture line, which is part of the axis y(Fig. 2). Fig. 3a shows the curve J for the break angle 30°, Fig. 2b shows the curve for the break angle 60°.



(a) *(b)*

Figure 3. *J-integral on the interval* $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$, *a is for the break angle* 30°, *b is for the break angle* 60°

As follows from the dependencies in Fig. 3 a and b dependences, the maximum values J for both angles are reached at the points lying on the fracture line, i.e., on the axis y. This means that possible crack growth will develop in the directions of the fracture line. As the fracture angle increased, the maximum J-integral increased and the minimum decreased. The effect of the fracture angle on the qualitative appearance of the J-integral distribution curve is insignificant.

Fig. 4 shows the crack opening in the fracture section $x = 0$. The opening is understood as the value of the difference of displacements corresponding to the upper and lower banks of the crack. Fig. 4a and 4b show the three-dimensional opening of half of the crack (points of different colors correspond to the upper and lower banks) for the 30° and 60° fracture angles. The solid curve shows the position of the points in the fracture section. As can be seen, the effect of the fracture angle on the opening is also insignificant.

Fig. 5 shows the opening of the entire crack in the section $y = 0$ (Fig. 5a corresponds to the fracture angle 30°, Fig. 5b to the fracture angle 60°). The above results allow us to conclude that the crack opening is mainly determined by pressure and weakly depends on the fracture angle.

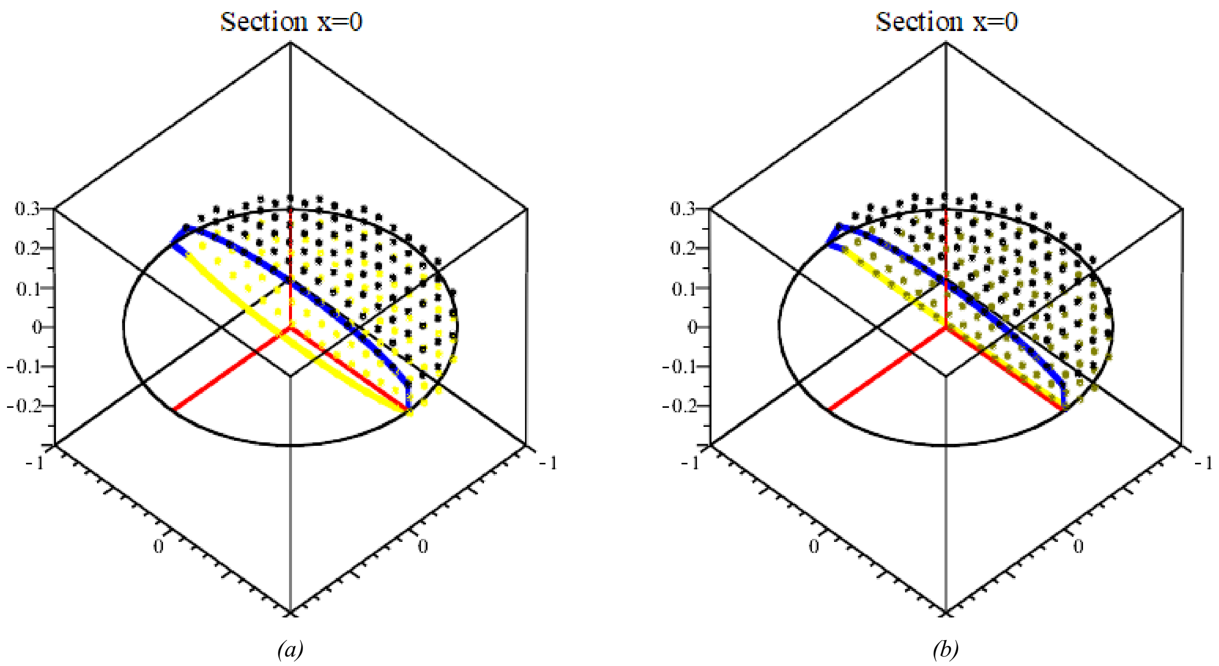


Figure 4. *Crack opening in space (a is the fracture angle* 30°*, b is the fracture angle* 60°*). The solid curve corresponds to the points of the section* $x = 0$

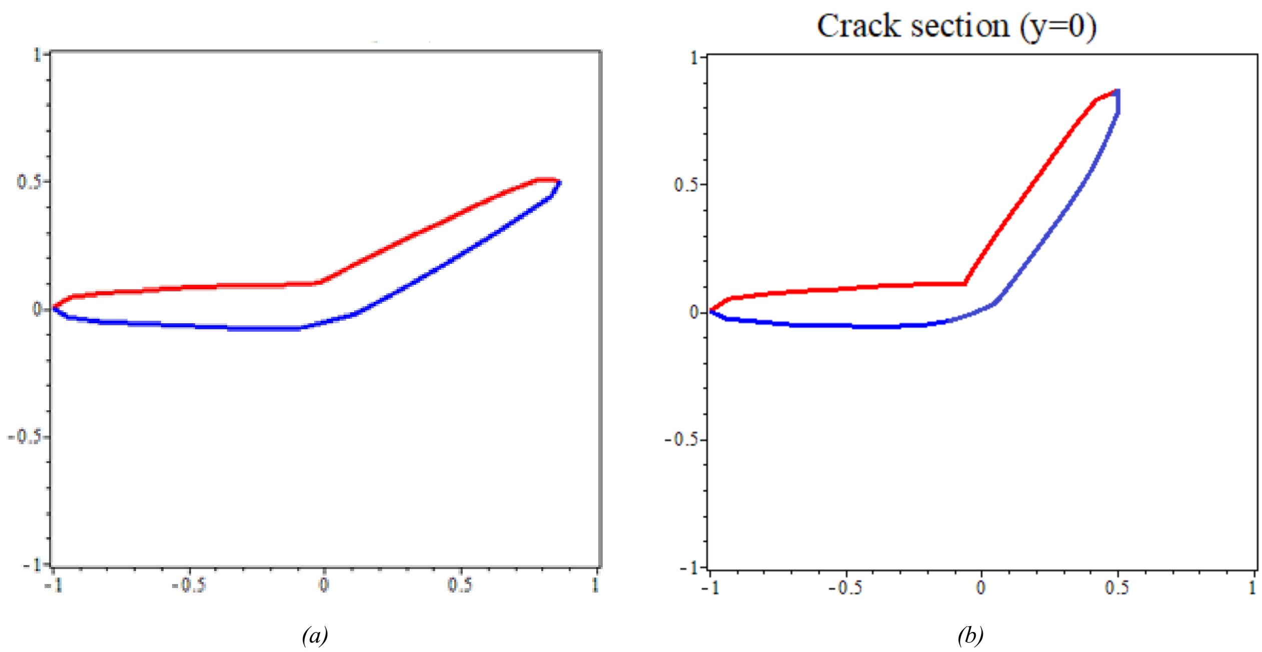


Figure 5. *Crack opening in the section* $y = 0$ *(a is the fracture angle* 30°*, b is the fracture angle* 60°*)*

The graphs (Figs. 4–5) show that the maximum opening is achieved in the fracture section $x = 0$. This corresponds to the results of the location of the maximum values of the J-integral and the directions of possible crack growth along the axis y.

It is known that in the vicinity of the crack boundary the stresses have a feature $\sigma \sim !/\sqrt{s}$, where $s-$ is the distance to the crack edge. To control the stresses, their calculation was made in the cylindrical coordinate system r,φ,z for angles $\varphi \in [\pi/2, 3\pi/2]$, corresponding to the boundary of the semicircle $z = 0, \quad x^2 + y^2 = R^2, \quad x \leq 0$. The value of radius was taken as the value of $R = 1.01$. This corresponds to the value of the distance s o the boundary equal to $s = 0.01$.

The graphs of stress distribution along the crack edge, depending on the angular coordinate ϕ in the polar coordinate system, are shown in Fig. 6 (a corresponds to the fracture angle 30°, b to the fracture angle 60°).

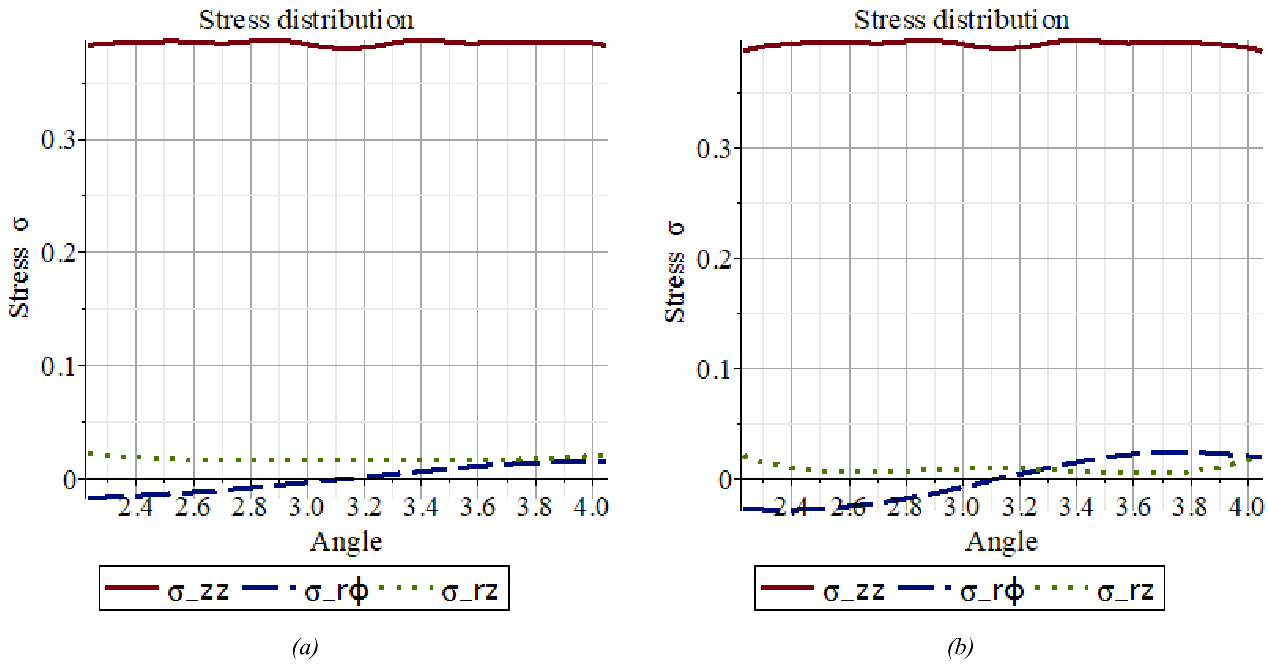


Figure 6. *Stress $\sigma_{zz},\sigma_{r\varphi},\sigma_{rz}$ vs. angular coordinate φ along the crack boundary in the cylindrical coordinate system (r,φ,z), $r = 1.01$, $\pi/2 \leq \varphi \leq 3\pi/2$, $z = 0$ (a is the fracture angle 30°, b is the fracture angle 60°)*

It can be seen (Fig. 6) that the stresses σ_{zz} do not depend on the fracture angle. The stresses differ greatly at σ_{rz}. That is, these calculations show that the growth of the J-integral when approaching the fracture is mainly provided by the growth of the stress σ_{rz}. Thus, the crack will grow in the direction of the fracture line if the fracture criterion is met.

To check the influence of the crack loading method, calculations have been made in which the crack sides are free from loads, with the elastic space subjected to tension by stresses acting at infinity along the axis z. That is, an elastic medium weakened by a fracture crack is considered. The angle of the fracture is known and is equal to 30 $\circ$. At infinity, a tensile load $\sigma = 0.1$, perpendicular to the plane of one of the fracture parts, is applied, as shown in Fig. 7.

Fig. 8 shows plots of the stress intensity coefficients K_I, $K_{II,}$ K_{III} respectively. The intensity coefficients were calculated by the displacement method using asymptotic formulas:

$$u_z = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}} sin\frac{\theta}{2}\left(2 - 2\vartheta - cos^2\frac{\theta}{2}\right)$$

$$u_x = \frac{K_{II}}{\mu}\sqrt{\frac{r}{2\pi}} sin\frac{\theta}{2}\left(2 - 2\vartheta + cos^2\frac{\theta}{2}\right)$$

$$u_y = \frac{K_{III}}{\mu}\sqrt{\frac{r}{2\pi}} sin\frac{\theta}{2}.$$

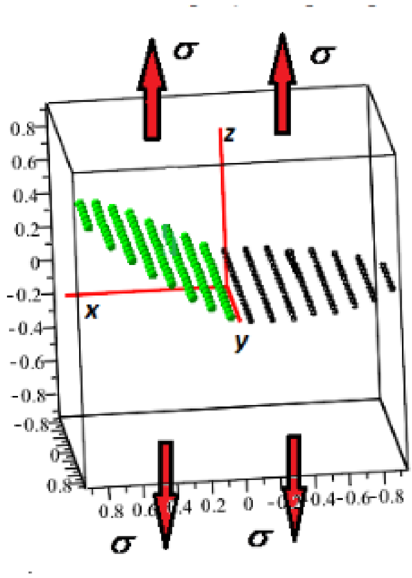
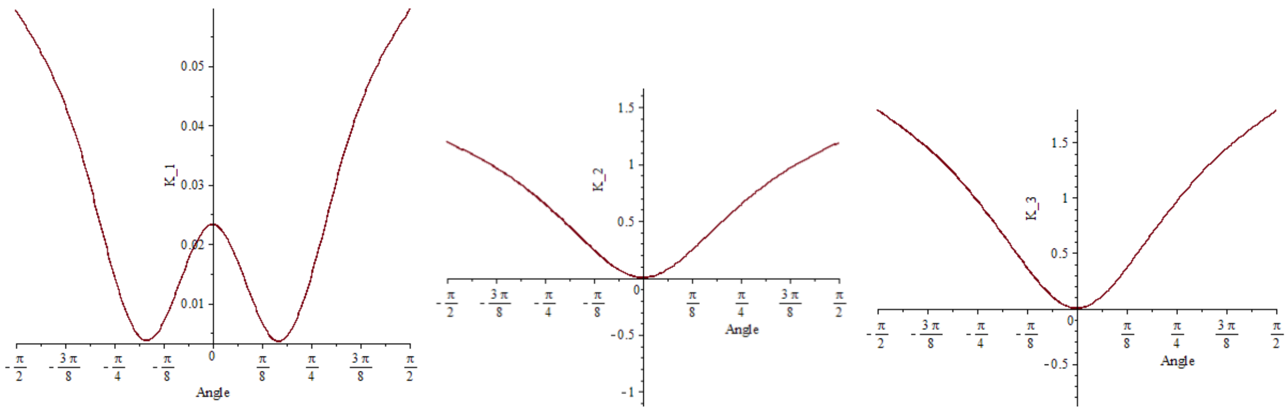


Figure 7. *Crack with fracture* 30°, *the load is applied in the direction of the OZ axis*

The graphs show the distribution of the corresponding coefficients on the interval $(-\pi/2,\pi/2)$. The zero angle corresponds to the fracture boundary point $x = -1, y = 0, z = 0$, the angles $(-\pi/2,\pi/2)$ correspond to the fracture points of the boundary.



(a) (b) (c)

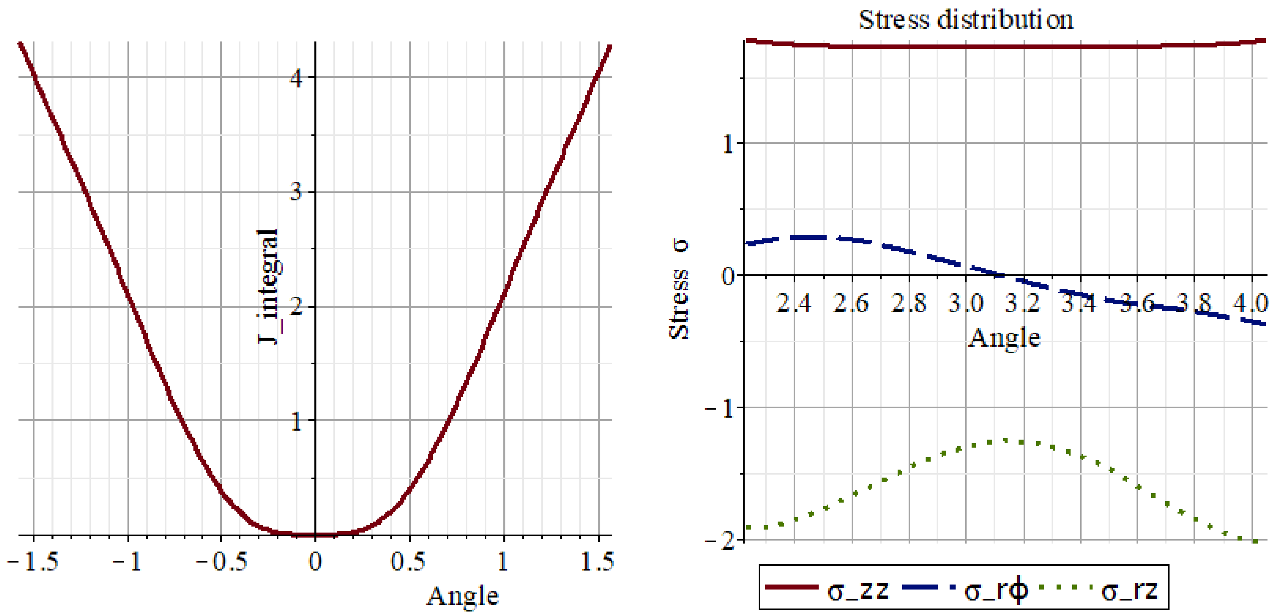
Figure 8. *Distribution of intensity coefficients: a –* K_I, *b –* K_{II} , *c –* K_{III}

J-integral and the stress distribution along the boundary of the crack segment perpendicular to the axis of z in the cylindrical coordinate system are presented in Figs. 9 a, b, respectively. As in the previous cases, the value of the distance to the crack edge was used $s = 0.01$.

The above calculations show a very strong dependence of the problem on the nature of the load.

Conclusions

1. Fracture cracks always start to grow along the fracture line, regardless of the type of load.
2. If the fracture crack is under pressure, its opening and stress distribution weakly depend on the fracture angle (they are slightly larger with a larger fracture angle).
3. The stresses are very much dependent on the nature of the crack load. If the fracture crack is passive and the load is applied at infinity, the stresses in the vicinity of the crack are very different.
4. The crack under load applied at infinity is less stable. This follows from a comparison of the value of the combination of stress intensity coefficients for a crack under pressure and a crack in space subjected to tension at infinity.



(a) (b)

Figure 9. *a – J-integral, b – stress distribution*

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